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Experimental Implementation of Fixed-Time Leader-Follower Axial Alignment Tracking

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***Abstract*—In this paper, the fixed-time leader-follower axial alignment tracking problem for a group of cooperative agents is investigated. The leader is dynamic and only transmits its position and velocity to its neighbors. A fixed-time algorithm is proposed to solve the consensus tracking problem. Each follower estimates the leader state in a fixed-time using distributed observers. To solve the consensus problem, based on the leader estimate, the followers collectively align their positions with the leader position in a fixed-time which does not depend on the initial positions. The experimental results show the effectiveness and robustness of the proposed fixed-time leader-follower consensus algorithm even in the presence of physical limitations such as packet loss, information delay, etc.**

***Index Terms*—multiple mobile robots, fixed-time stability, leader-follower consensus, distributed observer, ROS(Robot Operating System)**

I. Introduction

In recent years, many works on multiple robots have been done in many areas, e.g. formation control [1], [2], target tracking [3], [4], optimal coverage [5], distributed monitoring [6], [7], flocking [8], swarming [9], rendezvous [10], etc. Compare with a single robot, multiple robots may perform a mission more efficiently and provide higher flexibility during the task execution. One of the fundamental problems on multiple robots is consensus [11] to guarantee agreement from all agents regarding a certain quantity of interest via local interaction. In such missions, it is important to achieve coordination between robots without require their initial configurations [12].

The convergence rate analysis becomes an interesting research topic in the area of stabilization and leader following consensus [11]. In fact, the convergence rate is a significant performance index to validate the effectiveness of the control algorithms. Most of the existing results in leader following consensus concentrate on asymptotic [13] or finite-time convergence [14]. When the convergence is asymptotic, the tracking errors converge to zero when time approaches to infinity. For finite-time convergence, the tracking errors converge to zero in a finite time, but the settling time often depends on the initial conditions. Fixed-time stability has been proposed to define control algorithms which guarantee that the settling time is upper bounded regardless to the initial conditions [15]. Based on the sliding mode theory [16], some nonlinear switching controllers have been proposed to ensure fixed-time convergence [12].

In [12], the leader-follower consensus problem for nonholonomic mobile robots have been discussed. The convergence of the tracking errors is achieved in a finite time which does not depend on the initial conditions. In [17], a new method was proposed to design a tracking controller for one nonholonomic mobile robot such that the tracking errors converge to zero for any arbitrary initial tracking error in a fixed-time. For discrete time system, a decentralized model predictive protocol which uses the difference between two consecutive inputs is derived such that the consensus problem for multiple mobile robots is achieved [18] .

In this paper, we consider the fixed-time leader-follower axial alignment tracking problem for group of cooperative robots with double-integrator dynamics. Each follower estimates the leader state in a fixed-time using distributed observers. To solve the consensus problem, based on the leader estimate, a fixed-time controller is derived. Using the proposed controller, an upper bound of the settling time is provided regardless of initial conditions. Thereby, a decentralized observer-based control protocol is proposed for each agent to solve the leader-follower alignment problem in a fixed-time.

The paper is organized as follows. In Section II, the fixed-time concepts and graph theory are briefly reviewed. In Section III, the problem statement is formulated. In Section IV, the controller design which solves the fixed-time leader-follower axial alignment tracking problem is discussed for double integrator MAS. In section V, the experimental validation in real time using minilab robots and gazebo is presented. Finally, conclusions are given in section VI.

II. Recalls on fixed-time stability

Let us consider system

$$\begin{cases} \dot{x}(t) = F(t, x(t)) \\ x(0) = x_0, \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state, $F : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function and $F(t, 0) = 0$ for $t > 0$. The solutions of (1) are understood in the Filippov sense [19].

***Definition** 1:* [20] The origin of system (1) is a globally finite-time equilibrium if there is a function $T : \mathbb{R}^n \to \mathbb{R}^+$ such that for all $x_0 \in \mathbb{R}^n$, the solution $x(t, x_0)$ of system (1) is defined and $x(t, x_0) \in \mathbb{R}^n$ for $t \in [0, T(x_0))$ and $\lim_{t \to T(x_0)} x(t, x_0) = 0$. $T(x_0)$ is called the settling time function.

***Definition** 2:* [15] The origin of system (1) is a globally fixed-time equilibrium if it is globally finite-time stable and the settling time function $T(x_0)$ is bounded by a positive number $T_{max} > 0$, i.e. $T(x_0) \leq T_{max}$, $\forall x_0 \in \mathbb{R}^n$

***Lemma** 1:* [15] Assume that there exists a continuously differentiable positive definite and radially unbounded function $V : \mathbb{R}^n \to \mathbb{R}^+$ such that

$$\dot{V}(x) \leq -\alpha V^p(x) - \beta V^q(x) \quad (2)$$

with $\alpha > 0$, $\beta > 0$, $0 < p < 1$ and $q > 1$. Then, the origin of system (1) is globally fixed-time stable with settling time estimate

$$T(x_0) \leq T_{max} = \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)} \quad (3)$$

***Remark** 1:* [21] If $p = 1 - \frac{1}{\mu}$ and $q = 1 + \frac{1}{\mu}$ with $\mu \geq 1$, the settling time can be estimated by a less conservative bound:

$$T(x_0) \leq T_{max} = \frac{\pi\mu}{2\sqrt{\alpha\beta}} \quad (4)$$

A. Graph Theory

Let us consider a group of $N + 1$ robots with one leader and N followers. Among the N followers, the communication topology can be represented by graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V} = \{1, \ldots, N\}$ defines the set of nodes, corresponding to the followers, and $\mathcal{E} \subseteq \{\mathcal{V} \times \mathcal{V}\}$ defines the edge set. A link $(j, i) \in \mathcal{E}$, with $i \neq j$, exists if agent i receives information from its neighbor j. The adjacency matrix $A = (a_{ij}) \in \mathbb{R}^{N \times N}$ satisfies $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. The corresponding Laplacian matrix is given by $L = (l_{ij}) \in \mathbb{R}^{N \times N}$ with $l_{ii} = \sum_{j=1,\, j \neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. The links between the leader and the followers are characterized by matrix $B = diag(b_1, \ldots, b_N)$ where $b_i > 0$ if the leader state is available to follower i and where $b_i = 0$ otherwise.

In this paper, it is assumed that the communication topology among the N followers is undirected. It means that the adjacency matrix A is symmetric.

III. Problem Statement

Consider a multi-agent system consisting of a leader (which could be virtual) labeled by 0, and N followers, labeled by $i \in \{1, \ldots, N\}$. Here, we consider holonomic mobile robots moving in a two-dimensional plane. The axes of the workspace are shown in Fig. 1. Since we assume that motions along the x and y-axes are decoupled, the system dynamics can be modeled as

$$\begin{aligned} \dot{x}_{1,i}(t) &= x_{2,i}(t) \\ \dot{x}_{2,i}(t) &= u_{x,i}(t) \\ \dot{y}_{1,i}(t) &= y_{2,i}(t) \\ \dot{y}_{2,i}(t) &= u_{y,i}(t) \end{aligned} \quad (5)$$

where $x_{1,i}(t) \in \mathbb{R}$ and $y_{1,i}(t) \in \mathbb{R}$ are the position, $x_{2,i}(t) \in \mathbb{R}$ and $y_{2,i}(t) \in \mathbb{R}$ are the velocity, $u_{x,i}(t) \in \mathbb{R}$ and $u_{y,i}(t) \in \mathbb{R}$ are the control inputs along the x-axis and y-axis, respectively.

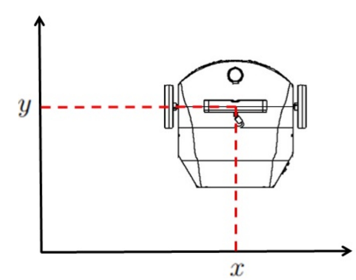


Fig. 1. Top view of the considered mobile robot.

***Remark** 2:* In the following, let us only consider the dynamics along the x-direction.

The leader dynamics is given by the following double-integrator system

$$\begin{cases} \dot{x}_{1,0}(t) = x_{2,0}(t) \\ \dot{x}_{2,0}(t) = u_{x,0}(t) \end{cases} \quad (6)$$

where $x_0 = [x_{1,0}, x_{2,0}]^T \in \mathbb{R}^2$ (resp. $u_{x,0} \in \mathbb{R}$) is the leader state (resp. leader control input) along the x-axis. The dynamics of the ith follower is as follows

$$\left\{ \begin{array}{rcl} \dot{x}_{1,i}(t) & = & x_{2,i}(t) \\ \dot{x}_{2,i}(t) & = & u_{x,i}(t) + d_i(t) \end{array} \right. \tag{7}$$

where $x_i = [x_{1,i}, x_{2,i}]^T \in \mathbb{R}^2$ (resp. $u_{x,i} \in \mathbb{R}$) is the state (resp. control input) of the ith follower. The unknown perturbation of the ith agent is given by $d_i \in \mathbb{R}$.

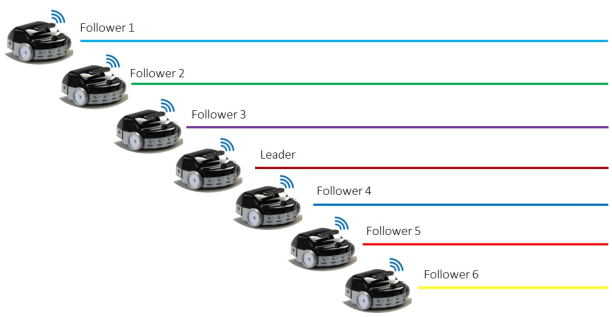


Fig. 2. Illustration of the fixed-time axial alignment tracking problem.

Fig. 2 provides an illustration of the fixed-time consensus tracking problem where only the dynamics along the x-direction are considered. To solve this problem, the following assumptions are made.

***Assumption** 1:* It is assumed that the communication topology among the N followers is undirected, fixed and connected. It means that the adjacency matrix A is symmetric. It is also assumed that there is at least one strictly positive parameter b_i.

***Assumption** 2:* The followers do not known the leader control input. Nevertheless, each neighboring agent knows its upper bounds $u_{x,0}$, defined as follows

$$|u_{x,0}(t)| \quad \leq \quad u_0^{max} \tag{8}$$

with $u_i^{max} \in \mathbb{R}^+$.

***Assumption** 3:* For each follower, the perturbation $d_i(t)$ is unknown but it is bounded as follows

$$|d_i(t)| \quad \leq \quad d_i^{max} \tag{9}$$

with $d_i^{max} \in \mathbb{R}^+$.

IV. Fixed-time Axial Alignment Tracking for Agents with Double-Integrator Dynamics

A. Fixed-time Observer

To estimate the leader state in a prescribed time, distributed observers are designed for each follower $i \in \{1, \ldots, N\}$. Indeed, the leader state is only available to its neighboring followers. Let us introduce the following observer as follows

$$\left\{ \begin{array}{rcl} \dot{\hat{x}}_{1,i} & = & \hat{x}_{2,i} \\ & & +\rho_1 \operatorname{sign}\left(\sum_{j=1}^N a_{ij}(\hat{x}_{1,j} - \hat{x}_{1,i}) + b_i(x_{1,0} - \hat{x}_{1,i})\right) \\ & & +\sigma_1 \left\lceil \sum_{j=1}^N a_{ij}(\hat{x}_{1,j} - \hat{x}_{1,i}) + b_i(x_{1,0} - \hat{x}_{1,i}) \right\rfloor^2 \\ \dot{\hat{x}}_{2,i} & = & \rho_2 \operatorname{sign}\left(\sum_{j=1}^N a_{ij}(\hat{x}_{2,j} - \hat{x}_{2,i}) + b_i(x_{2,0} - \hat{x}_{2,i})\right) \\ & & +\sigma_2 \left\lceil \sum_{j=1}^N a_{ij}(\hat{x}_{2,j} - \hat{x}_{2,i}) + b_i(x_{2,0} - \hat{x}_{2,i}) \right\rfloor^2 \end{array} \right. \tag{10}$$

where $\hat{x}_{k,i}$ $(k = \{1, 2\})$ is the estimation of the leader state $x_{k,0}$ for the ith follower, ρ_k and σ_k are positive constants, which will be given hereafter.

The fixed-time stabilization of the estimation errors

$$\tilde{x}_{k,i} = \hat{x}_{k,i} - x_{k,0} \qquad (i = \{1, \ldots, N\},\ k = \{1, 2\}) \tag{11}$$

is introduced in the following theorem.

***Theorem** 1:* Suppose that Assumptions 1-2 are satisfied. If the gains of the distributed observer (10) verify

$$\left\{ \begin{array}{rcl} \sigma_k & = & \frac{\epsilon\sqrt{N}}{(2\lambda_{min}(L+B))^{\frac{3}{2}}}, \forall k = 1, 2 \\ \rho_1 & = & \epsilon\sqrt{\frac{\lambda_{max}(L+B)}{2\lambda_{min}(L+B)}} \\ \rho_2 & = & u_0^{max} + \epsilon\sqrt{\frac{\lambda_{max}(L+B)}{2\lambda_{min}(L+B)}} \end{array} \right. \tag{12}$$

with $\epsilon > 0$, then, for any initial condition, the estimation errors (11) converge to zero. An upper bound of the convergence time can be given as

$$T_o = \frac{2\pi}{\epsilon} \tag{13}$$

Proof. Using (10), the dynamics of the observation error is given by

$$\begin{array}{rcl} \dot{\tilde{x}}_{1,i} & = & \tilde{x}_{2,i} + \rho_1 \operatorname{sign}\left(\sum_{j=1}^N a_{ij}(\tilde{x}_{1,j} - \tilde{x}_{1,i}) - b_i\tilde{x}_{1,i}\right) \\ & & +\sigma_1 \left\lceil \sum_{j=1}^N a_{ij}(\tilde{x}_{1,j} - \tilde{x}_{1,i}) - b_i\tilde{x}_{1,i} \right\rfloor^2 \\ \dot{\tilde{x}}_{2,i} & = & \rho_2 \operatorname{sign}\left(\sum_{j=1}^N a_{ij}(\tilde{x}_{2,j} - \tilde{x}_{2,i}) - b_i\tilde{x}_{2,i}\right) \\ & & +\sigma_2 \left\lceil \sum_{j=1}^N a_{ij}(\tilde{x}_{2,j} - \tilde{x}_{2,i}) - b_i\tilde{x}_{2,i} \right\rfloor^2 - u_{x,0} \end{array} \tag{14}$$

Let us denote

$$\tilde{x}_k = [\tilde{x}_{k,1}, \ldots, \tilde{x}_{k,N}]^T \tag{15}$$

Then, for $\tilde{x}_1$ and $\tilde{x}_2$, one can obtain

$$\dot{\tilde{x}}_1 = \tilde{x}_2 - \rho_1 \operatorname{sign}((L+B)\tilde{x}_1) - \sigma_1 \lceil (L+B)\tilde{x}_1 \rfloor^2 \tag{16}$$

$$\dot{\tilde{x}}_2 = -\rho_2 \operatorname{sign}((L+B)\tilde{x}_2) - \sigma_2 \lceil (L+B)\tilde{x}_2 \rfloor^2 - \mathbf{1}u_{x,0} \tag{17}$$

We complete the proof by two steps.

- Let us consider system (17). Consider the candidate Lyapunov function for subsystem (17): $V_1 = \frac{1}{2}\tilde{x}_2^T(L+B)\tilde{x}_2$. Its time derivative along (18) is given by

$$\dot{V}_1 \quad \leq \quad -\epsilon V_1^{\frac{1}{2}} - \epsilon V_1^{\frac{3}{2}}$$

  Using Lemma 1, one can prove that $\tilde{x}_2$ converges to zero in a finite-time bounded by $\frac{\pi}{\epsilon}$.

- After $\tilde{x}_2$ converges to zero (i.e. when $t \geq \frac{\pi}{\epsilon}$), the dynamics of $\tilde{x}_1$ becomes

$$\dot{\tilde{x}}_1 = -\rho_1 \operatorname{sign}\left((L+B)\tilde{x}_1\right) - \sigma_1 \left\lceil (L+B)\tilde{x}_1 \right\rfloor^2 \quad (18)$$

Similarly to the previous step, the $\tilde{x}_1$ dynamics converges to zero. Indeed, consider the candidate Lyapunov function for system (18): $V_2 = \frac{1}{2}\tilde{x}_1^T(L+B)\tilde{x}_1$. Its time derivative along (18) is given by

$$\dot{V}_2 \leq -\epsilon V_2^{\frac{1}{2}} - \epsilon V_2^{\frac{3}{2}}$$

Hence, one can conclude that $\tilde{x}_1$ converges to zero and after that $\tilde{x}_2$ converges to zero in a finite-time bounded by $2\frac{\pi}{\epsilon}$.

Hence, one can conclude that the estimation errors (11) converge to zero in a fixed-time bounded by T_o. ■

B. Fixed-time Axial Alignment Tracking Controller

From Theorem 1, one can conclude that $\hat{x}_i = [\hat{x}_{1,i}, \hat{x}_{2,i}]^T = x_0$ for all $t \geq T_o$. Hence, after time T_o, each follower is able to indirectly access to the state of the leader and uses the estimate $\hat{x}_i$ in the consensus protocol.

Let us denote the tracking errors as follows ($i = \{1,2\}$, $k = \{1,2\}$)

$$e_{k,i} = x_{k,i} - \hat{x}_{k,i} = x_{k,i} - x_{k,0} - \tilde{x}_{k,i} \quad (19)$$

From (6)-(7) and using Theorem 1, for each follower $i = \{1,\ldots,N\}$ and for all $t \geq T_o$, the tracking error dynamics becomes

$$\begin{aligned} \dot{e}_{1,i} &= e_{2,i} \\ \dot{e}_{2,i} &= u_{x,i} + d_i - u_{x,0} \end{aligned} \quad (20)$$

It is clear that (20) is a second-order system. To deal with the observer-based alignment tracking problem, for each follower $i = \{1,\ldots,N\}$, the control objective is to design $u_{x,i}$ such that the origin of system (20) is fixed-time stable with the settling time estimate T in spite of the presence of matched perturbations.

***Theorem** 2:* Let us consider the leader-follower system (6)-(7). Suppose that Assumptions 1-3 are satisfied and the gains of the distributed observer (10) verify (12). The leader-follower axial alignment problem is solved in a fixed-time using the decentralized controllers

$$u_{i,x} = \begin{cases} 0, \ \forall t < T_o \\ -\frac{\alpha_1 + 3\beta_1 e_{1,i}^2 + 2a_i}{2} \operatorname{sign}(s_i) - \left\lfloor \alpha_2 s_i + \beta_2 \lfloor s_i \rceil^3 \right\rceil^{\frac{1}{2}}, \ t \geq T_o \end{cases} \quad (21)$$

with the sliding surface

$$s_i = e_{2,i} + \left\lfloor \lfloor e_{2,i} \rceil^2 + \alpha_1 e_{1,i} + \beta_1 \lfloor e_{1,i} \rceil^3 \right\rceil^{\frac{1}{2}} \quad (22)$$

where α_1, α_2, β_1 and β_2 are positive constants, a_i is a positive constant given hereafter. The settling time is explicitly defined as

$$T = T_o + \frac{2}{\sqrt{\alpha_2}} + \frac{2}{\sqrt{\beta_2}} + \frac{2\sqrt{2}}{\sqrt{\alpha_1}} + \frac{2\sqrt{2}}{\sqrt{\beta_1}} \quad (23)$$

Proof. We complete the proof by two steps.

- Following [15], let us consider the candidate Lyapunov function $V_3 = |s_i|$. Its derivative is,

$$\dot{V}_3 = \dot{e}_{2,i} \operatorname{sign}(s_i) + \frac{|e_{2,i}|\dot{e}_{2,i} \operatorname{sign}(s_i) + \frac{\alpha_1 + 3\beta_1 e_{1,i}^2}{2} e_{2,i} \operatorname{sign}(s_i)}{|\lfloor e_{2,i} \rceil^2 + \alpha_1 e_{1,i} + \beta_1 \lfloor e_{1,i} \rceil^3|^{\frac{1}{2}}} \quad (24)$$

Setting

$$a_i \geq d_i^{max} + u_0^{max} \quad (25)$$

one can conclude that

$$\dot{V}_3 \leq -(\alpha_2 V_3 + \beta_2 V_3^3)^{\frac{1}{2}} \quad (26)$$

From Lemma 1, it is clear that $s_i = 0$ when $t \geq T_o + \frac{2}{\sqrt{\alpha_2}} + \frac{2}{\sqrt{\beta_2}}$.
- The sliding dynamics ($s_i = 0$) can be expressed as

$$\dot{e}_{1,i} = -\left\lfloor \frac{\alpha_1 e_{1,i} + \beta_1 \lfloor e_{1,i} \rceil^3}{2} \right\rceil^{\frac{1}{2}} \quad (27)$$

Using the candidate Lyapunov function $V_4 = |e_{1,i}|$, one can obtain

$$\dot{V}_4 = -(\frac{\alpha_1}{2} V_4 + \frac{\beta_1}{2} V_4^3)^{\frac{1}{2}} \quad (28)$$

It is clear using Lemma 1 that $e_{1,i} = 0$ when $t \geq T_s$. Furthermore, since $e_{1,i} = 0$ and $s_i = 0$, then $e_{2,i} = 0$.

This concludes the proof. ■

V. Experimental Validation

A. Experimental Platform

This section briefly introduces the experimental platform Minilab Robot, available at LAMIH, UPHF. The experiments were performed on a group of mobile robots supplied by Enova Robotics to test and validate the effectiveness of the theoretical results given in the previous section. Fig. 3 shows the architecture of the experimental platform used in an indoor environment. In this process, control algorithms are programmed using ROS (Robotic Operating System) with Gazebo-ROS as a reality virtual simulation.

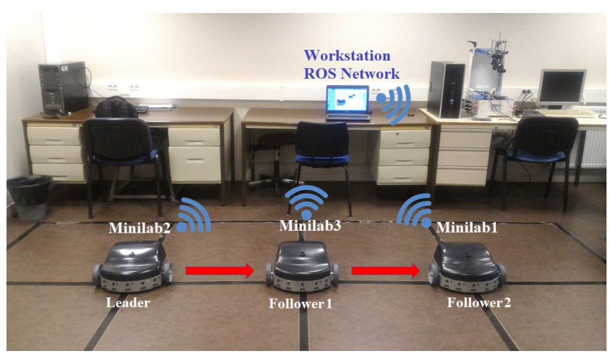


Fig. 3. Illustration of the experimental setup.

Generally, in ROS environment, we can perform simulation for several robots using only one workstation. Instead of using multiple workstations, in the following we introduce the idea of gazebo simulator into real robots to reduce cost due to the use of several workstations. Since we are working on the consensus problem for MAS using only one workstation, we need to appropriately select the wifi configuration and design multiple master ROS in a decentralized architecture for multiple robots [22].

B. Implementation of Fixed-time Axial Alignment Tracking Controller on Minilab Platform

Fig. 4 shows the communication topology for the leader-follower MAS scenario in ROS-Gazebo. A MAS with $N = 6$ followers labeled by $1 - 6$ and one leader labeled by 0 is considered. One can see that the communication topology is fixed and connected. It is characterized by the following Laplacian L and the matrix B which describes links between the leader and the followers given as follows

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From matrix B, it is clear that agents 2, 3, 4 and 6 do not have direct link with agent 0. Assumption 1 is satisfied.

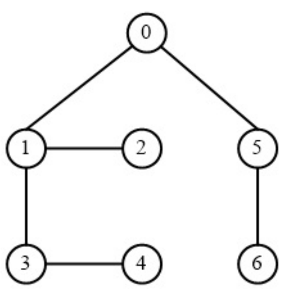


Fig. 4. Topology of MAS using ROS.

C. Experimental Results

1) Fixed-time Trajectory Tracking: Let us first only consider one leader and one follower. For this scenario, the control objective is reduced as a fixed-time trajectory tracking problem, i.e. system (7) follows the desired trajectory $x_d = x_0$.

The initial state of the robot is $x = [x_1 = -1.5, x_2 = 0]^T \in \mathbb{R}^2$. The settling time is set as $T = 3.05s$. The desired trajectory is generated by (6) with $x_0 = x_d(0) = 0$, $u_0 = u_d = 0.2$.

Using Theorem 2, the tracking controller (21) guarantees the stabilization of the tracking errors to the origin in a finite-time bounded by $T = 3.05s$. Fig. 5(a) shows that the actual state trajectory x accurately tracks the desired state x_d at $1.2s$ for the first state and from Fig. 5(b) for the second state at $1.25s$. Hence, the origin of the closed-loop system is globally finite-time stable. Furthermore, since T does not depend on the initial states, the proposed protocol is a fixed-time controller. Fig. 5(c) shows the control inputs for linear x acceleration.

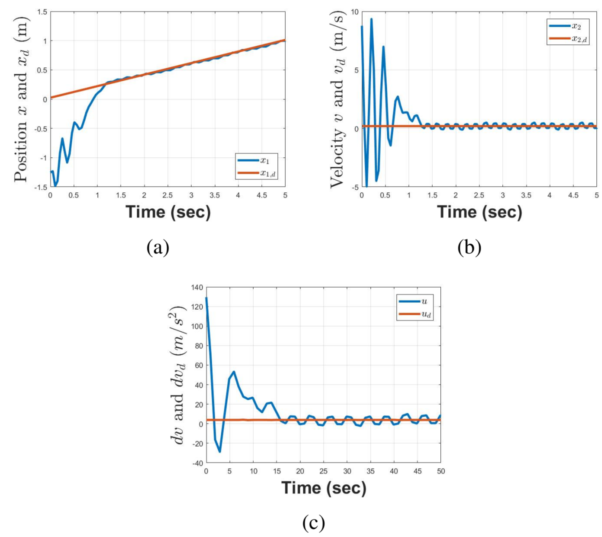


Fig. 5. Experimental results for the fixed-time trajectory tracking problem: Time response of actual state trajectory x and desired state trajectory x_d with constant linear desired control input.

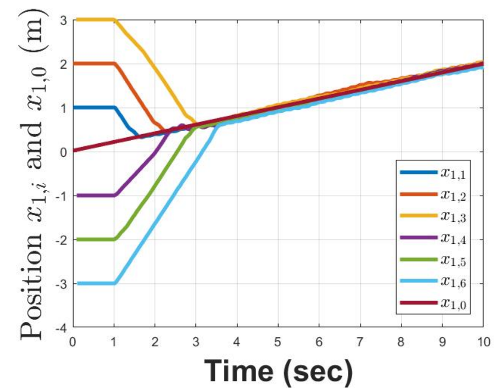
2) Fixed-time Axial Alignment: The control objective is that the six followers (7), track the leader (6) in a fixed-time using only local exchanged information.

The initial position of the robots is given by the following vector $x(0) = [3, 2, 1, -1, -2, -3]^T$ where the initial velocity is zero. Recall that T_o is the time needed for an agent to estimate the leader state (prescribed time observation). It should be noted that the estimation in (13) depends on the gains of observer. The settling time is explicitly defined as $T = T_o + \frac{2}{\sqrt{\alpha_2}} + \frac{2}{\sqrt{\beta_2}} + \frac{2\sqrt{2}}{\sqrt{\alpha_1}} + \frac{2\sqrt{2}}{\sqrt{\beta_1}}$.

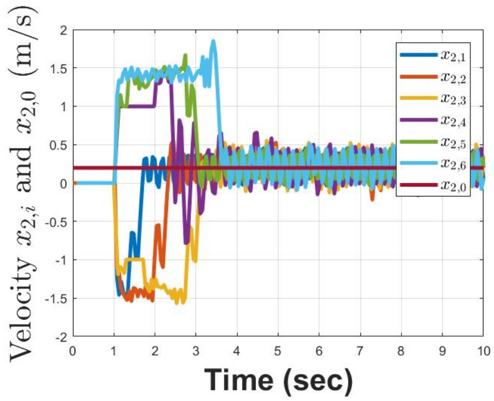
The control parameter are selected as: $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 10$ for controller (21).

The desired trajectory for the leader is generated by (6) with $x_{1,0}(0) = 0$, $x_{2,0}(t) = 0.2$. Hence, we set $a = 1$.

Using Theorem 1, the distributed observer (10) guarantees the stabilization of the estimation errors to the origin in a finite-time bounded by $T_o = 1s$. The distributed observers accurately reconstruct the leader state for each robot before T_o. Using Theorem 2, the consensus controller guarantees the stabilization of the errors to the origin in a finite-time bounded by $T = 6.55s$. Fig. 6 shows that the actual state trajectory accurately tracks the leader state before T in spite of the presence of disturbances and uncertainties inherent to the experimental setup. One can conclude that using the proposed controller, the leader-follower axial alignment is achieved in a prescribed time. The origin of the closed-loop system is globally finite-time stable contrary to existing controllers which only provide semi-global finite-time stability property. Furthermore, since T does not depend on the initial states of robots, the proposed protocol is distributed.



(a)



(b)

Fig. 6. Experimental results: Time response of the actual state trajectory of the followers and of the leader

VI. Conclusion and Future Work

We have investigated the fixed-time leader-follower axial alignment tracking problem for a group of cooperative agents. Distributed observers have been designed to estimate the leader state. Some sufficient conditions have been established for the observer and controller gains in terms of graph connectivity to achieve consensus. The effectiveness of the theoretical results has been validated through experimental results. For the future work, implementation on nonholonomic unicycle-type model will be considered.

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